PHYSICAL REVIEW VOLUME 132, NUMBER 5 1 DECEMBER 1963

Scattering of Protons from C^{12} in the Energy Range of 5–6 MeV*

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(Received 24 April 1963)

Excitation functions and angular distributions were obtained in the bombarding energy range 5–6 MeV for the elastic and inelastic (4.43-MeV state) scattering of protons from C^{12}, for the 4.43-MeV gamma rays. Three resonances were observed, at bombarding energies of 5.38, 5.68, and 5.90 MeV (excitation energies in N^{13} of 6.91, 7.19, and 7.40 MeV). The corresponding center-of-mass widths are 115±5 keV, 9±0.5 keV, and 75±5 keV, respectively. The center-of-mass elastic widths are 4.6, 0.36, and 6.9 keV, respectively. Analysis of the interference characteristics of the elastic scattering cross sections, together with analysis of the various angular distributions and applications of the Wigner limit rule, leads to the following assignments of quantum numbers: for the 5.38-MeV (bombarding energy) resonance, $3/2^+$; for the 5.68-MeV resonance, very probably $7/2^+$; for the 5.90-MeV resonance, $5/2^-$.

1. INTRODUCTION

The single-particle model of the nucleus promised to be quite useful in describing the excited states of light nuclei. It assumes a deep central potential well in which the nucleons form shells, filling them in order of $1s$, $1p$, $1d$, $2s$, etc. This simple scheme is then perturbed by the presence of residual two-body forces when there is more than one nucleon outside the closed shell. These perturbation effects are first exhibited in the $1p$ shell ($A=5$–16).

As early as 1937 Hartree–Fock calculations of the levels of nuclei filling the $1p$ shell were done by Feenberg and Wigner,[1] and Feenberg and Phillips[2] using the L-S coupling scheme. In 1952, Kurath[3] used the j-j coupling scheme on the same nuclei. It was soon evident that neither of these two schemes gave satisfactory agreement with experiment.

To improve the agreement between the theory and experiment, Inglis[4] suggested that an intermediate coupling scheme be used. Kurath[5] made calculations on such a basis. The calculations considered the single-particle wave function in an harmonic oscillator well with individual l and s coupled to give j. These functions were then combined into a many-particle wave function of Hartree–Fock type with total angular momentum J and isotopic spin T.

In Kurath's calculations,[5] the nuclear parameters came from the radial part of the harmonic oscillator function

$$r\exp[-(r/r_p)^2]; \tag{1.1}$$

the usual spin-orbit term

$$a\mathbf{l}\cdot\mathbf{s}, \tag{1.2}$$

where a, the strength of the spin-orbit coupling, is the adjustable parameter; and the radial part of the central two-body interaction

$$A_0\exp[-(r_{12}/r_0)^2]. \tag{1.3}$$

The effects of the central two-body interaction are expressed in terms of the direct integral L and the exchange integral K.[1,4,5] These integrals are functions of the strength of the two-body interaction A_0, and the parameter

$$\delta=r_p/r_0 \tag{1.4}$$

that measures the ratio of the harmonic oscillator potential well radius to the range of nuclear two-body force.

Kurath's results[5] are expressed for a particular value of L/K (taken to be 6.8). The parameters are then a/K, which measures the relative strength of spin-orbit term with respect to central two-body energy interaction, and K which can be adjusted to match the experimental scale. For the case of $A=13$ only states of negative parity ($1p$ orbitals) were considered.

The positive parity states in $A=13$ nuclei were considered by Baker,[6] who used a model due to Lane,[7] according to which the low-lying positive parity states are formed by weakly coupling a $2s$- or $1d$-nucleon to the lowest shell model states of C^{12}, assumed not to be polarized by the additional nucleon. One of the results of this calculation is that there should be a $7/2^+$ state at an excitation of ~7.8 MeV in C^{13}, and in its mirror nucleus N^{13}.

A strong resonance was previously observed in $C^{12}+p$ reaction which corresponds to 7.40-MeV excitation energy in N^{13}.[8–11] Reich *et al.*[12] suggested $j=5/2^+$ for this state by extrapolating their data beyond the energy range they measured. A determination of the gamma

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[1] E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937).
[2] E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).
[3] D. Kurath, Phys. Rev. **88**, 804 (1952).
[4] D. Inglis, Rev. Mod. Phys. **25**, 309 (1953).
[5] D. Kurath, Phys. Rev. **101**, 216 (1956).
[6] F. C. Barker, Nucl. Phys. **28**, 96 (1961).
[7] A. M. Lane, Rev. Mod. Phys. **32**, 519 (1960).
[8] M. Martin, H. Schneider, and M. Sengert, Helv. Phys. Acta **26**, 595 (1953).
[9] B. Maeder, M. Martin, R. Miller, and H. Schneider, Helv. Phys. Acta **27**, 166 (1954).
[10] H. Schneider, Helv. Phys. Acta **29**, 55 (1956).
[11] C. P. Browne and J. R. Lamarsh, Phys. Rev. **104**, 1099 (1956).

angular distribution from $C^{12}(p,p'\gamma)C^{12}$ performed in this laboratory[13] tentatively confirmed this. This spin assignment was obtained by fitting the angular distribution of γ's at the 5.90-MeV resonance. The parity assignment was obtained from the fact that in the energy region where the 5.90 resonance has to be combined with the $3/2^+$ resonance at 5.38 MeV the angular distribution of gamma rays is symmetric about 90° c.m., which implies that both states have the same parity. However, this argument cannot be considered as final, since the superposition angular distribution can be symmetric about 90° c.m. by "accident" even though the two states have different parities.

Therefore, the investigation of the reaction following the bombardment of C^{12} by protons was carried out with two main purposes in mind: (1) to establish firmly the parameters of 5.90-MeV resonance and thus determine if this could correspond to the "missing" $5/2^-$ state in N^{13}, and (since a $7/2^+$ state can be formed only by incoming f wave protons on C^{12}, a resonance corresponding to the formation of this state is expected to be weak and narrow), (2) to make a careful search for weak and narrow resonances in this energy range.

2. APPARATUS

A beam of accelerated protons was obtained from the Columbia University Pegram Nuclear Physics Laboratories Van de Graaff accelerator. The magnetic field of the 90° deflection magnet, the entrance and exit apertures, together with the feedback on the accelerator, defined and maintained the energy of the beam to within $\pm 0.1\%$. The deflecting magnetic field, and thus the energy, was measured by the frequency of nuclear magnetic resonance for Li^7.

The γ rays were observed in a cylindrical aluminum scattering chamber, 9 in. in diameter and 6.5 in. high. The construction of 3/8-in. thick walls of the chamber was such that the absorption of γ rays was independent of the angle of observation. Carbon targets on thick backings were mounted on an electrically insulated post which passed through a rotating vacuum seal centrally located in the bottom of the chamber. Thus the targets were insulated from the rest of the chamber and were used to collect beam charge and provide a measure of the number of incident protons. The chamber entrance pipe contained the beam collimator together with the collimator scattering suppressor.

The detector was a 3-×3-in. NaI(Tl) crystal mounted on a 3-in. Du Mont photomultiplier tube type 6363 (10 stages). The tube and the crystal were first shielded by a μ-metal antimagnetic shield and then put in a γ-ray shield consisting of 2-in. lead around the crystal. The detector was mounted on a turntable and was outside the chamber. By observing the radiation from an isotropic source, the axis of the detector turntable was adjusted to pass through the point of beam impact on the target. This adjustment was made in such a way that the maximum deviation from isotropic distribution was 2% and the average deviation 1%. The angles were measured with a precision of $\pm 0.5°$. The detector assembly was mounted so that the face of the crystal was 15 in. from the target, subtending an angle of 11°.

The target itself was a thick deposit of carbon on a 10-mil tantalum disk. Its uniformity was checked microscopically. Since we were observing only the 4.43-MeV radiation from $C^{12}(p,p',\gamma)C^{12}$ reaction, and since we were interested only in the shapes of angular distributions and relative yields in the excitation curve, no determination of the target thickness was made.

The scattering chamber[14] used for observation of protons, both elastically and inelastically scattered, was a block of aluminum with a cylindrical cavity. The detector assembly was mounted on the lid. The lid could be rotated under vacuum, and the detector angles could be read with a precision of $\pm 0.1°$.

The detector was a 9-mil-thick CsI(Tl) crystal mounted via a Lucite light pipe on a Du Mont 6291 photomultiplier. The axis of the detection system was pointed at the center of the chamber and made an angle of 14°50′ with the horizontal plane. Consequently, the angle (ϕ) that the lid makes with the incident beam is not the laboratory angle (ψ) between the beam and the detector, but rather that ψ is calculated from[14]:

$$\cos\psi = \cos(14°50') \cos\phi = 0.96667 \cos\phi. \quad (2.1)$$

The detector-defining slit system subtended a solid angle of 1.242×10^{-3} sr.

The beam was admitted into the chamber through a collimator consisting of two defining slits 1/16 in. in diameter followed by an antiscattering slit 3/32 in. in diameter. After passing through the chamber, the beam was collected by a Faraday cup. To eliminate the γ background produced by the beam impact, the Faraday cup was placed 3 ft behind the chamber. The secondary electrons were electrostatically suppressed. The ionization of residual gas was eliminated by using auxiliary diffusion pumps to maintain high vacuum in the chamber and the Faraday cup.

The target consisted of a self-supporting carbon film mounted on a tantalum ring. The angle that the plane of the target made with the beam could be measured and adjusted at will. Target thickness was measured by considering the Coulomb scattering at 1.5-MeV bombarding energy, and it was determined to be 40 μg cm^{-2} which corresponds approximately to a proton energy loss of 2.8 keV at 5 MeV.

A small amount of hydrogen and oxygen contamination was present. The hydrogen contamination had no

[12] C. W. Reich, G. C. Phillips, and J. L. Russell, Jr., Phys. Rev. **104**, 143 (1956).

[13] L. J. Lidofsky, J. Weil, R. D. Bent, and K. W. Jones, Bull. Am. Phys. Soc. **2**, 29 (1957).

[14] H. Smotrich, Ph.D. dissertation, Columbia University, 1961 (unpublished). (The method of centering of the detector assembly is described in detail.)

influence on elastically scattered proton cross section determination. However, because of the hydrogen contamination, it was impossible to observe inelastically scattered protons at certain angles. The O^{16} contamination was determined by using a 180° magnetic spectrometer with a momentum resolution better than 0.2%[15]; it was found to be 1.3% by number of atoms. That correction, together with the known abundance of C^{13} (1%), was taken into account in determining the differential cross section for elastically scattered protons.

For bombarding energies close to the threshold (4.80-MeV bombarding energy), it was difficult to identify pulses from inelastically scattered protons in the presence of background. It was found that a foil ($\sim$26 μg cm^{-2}) of Formvar (Polyvinyl Formal) placed in front of the detector slits filtered out a substantial amount of background which was presumably due to heavy recoil ions. Such a foil was always used when inelastic protons were observed.

3. GAMMA RAYS FROM THE REACTION $C^{12}(p,p'\gamma)C^{12}$

A. Results

C^{12} has a $j^{\pi}=2^{+}$ excited state at 4.43 MeV.[16] If the energy of incident protons is larger than 4.80 MeV in laboratory, the resultant compound nucleus of N^{13} has then two open channels for proton decay: either it can go directly to 0^{+} ground state of C^{12} by elastically scattering a proton, or by inelastically scattering a proton to 2^{+} excited state of C^{12} which later decays into the ground state by emission of γ rays. These γ rays are considered first, the elastic and inelastic protons being the topic of subsequent sections.

Since the transition involved is a $2^{+}\rightarrow 0^{+}$ one, the emitted radiation is an electric quadrupole ($E2$). The largest angular momentum involved is $l=2$. Hence, if the angular distribution of γ rays is written as

$$W(\theta)=\sum_{n} A_n P_n(\cos\theta) \qquad (3.1)$$

then the highest index n for Legendre polynomials has to be 4.[17] Furthermore, as is shown later, the coefficients A_n for odd n's in (3.1) turns out to be zero, so that (3.1) can be written as

$$W(\theta)=1+a_2P_2+a_4P_4. \qquad (3.2)$$

Since

$$d\sigma/d\theta=K(1+a_2P_2+a_4P_4), \qquad (3.3)$$

it follows that

$$\sigma=\int (d\sigma/d\Omega)d\Omega=4\pi K. \qquad (3.4)$$

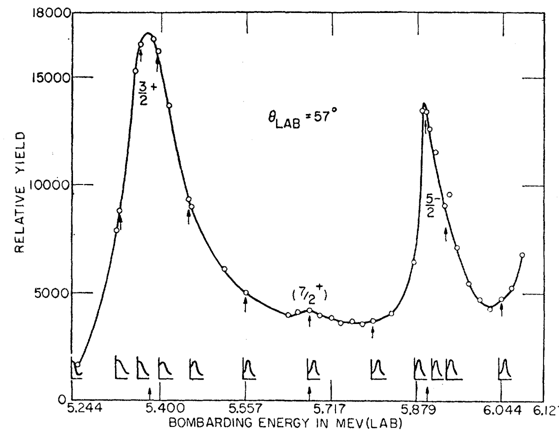

FIG. 1. Dependence on laboratory bombarding energy of the relative yield of 4.43-MeV γ rays from the reaction $C^{12}(p,p'\gamma)C^{12}$, at $\theta_{lab}=57°$. The small inserts indicate the shape between 0° and 90° c.m. of the gamma angular distributions at various energies.

At 57° in the laboratory, $P_2=0$, and, as is seen later, close to the resonance at 5.38-MeV bombarding energy $a_4\ll 1$, so that $(d\sigma/dr)(57°)\sim\sigma/4\pi$. The excitation function at this angle closely approximates the behavior of the total cross section.

Figure 1 represents the excitation curve taken at 57° with respect to the incident beam. The general features

TABLE I. The angular distributions of gamma rays.

at 5.25 MeV	$W=P_0+(0.392\pm0.015)P_2+(0.332\pm0.018)P_4$
at 5.33 MeV	$W=P_0+(0.449\pm0.014)P_2+(0.026\pm0.020)P_4$
at 5.33 MeV	$W=P_0+(0.467\pm0.014)P_2+(0.029\pm0.020)P_4$
at 5.36 MeV	$W=P_0+(0.457\pm0.037)P_2-(0.113\pm0.045)P_4$
at 5.40 MeV	$W=P_0+(0.435\pm0.041)P_2-(0.271\pm0.058)P_4$
at 5.56 MeV	$W=P_0+(0.521\pm0.027)P_2-(0.684\pm0.030)P_4$
at 5.68 MeV	$W=P_0+(0.372\pm0.074)P_2-(0.828\pm0.038)P_4$
at 5.80 MeV	$W=P_0+(0.513\pm0.029)P_2-(0.878\pm0.034)P_4$
at 5.88 MeV	$W=P_0+(0.508\pm0.019)P_2-(0.664\pm0.026)P_4$
at 5.90 MeV	$W=P_0+(0.488\pm0.021)P_2-(0.666\pm0.028)P_4$
at 5.94 MeV	$W=P_0+(0.509\pm0.025)P_2-(0.877\pm0.030)P_4$
at 6.05 MeV	$W=P_0+(0.417\pm0.033)P_2-(0.997\pm0.037)P_4$

[15] M. Tatcher (private communication).
[16] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. 11, 1 (1959).
[17] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons Inc., New York, 1952), p. 535 ff.

of the excitation curve, that is, the large resonances at 5.38- and 5.90-MeV bombarding energy, have been observed before in this laboratory[13] as well as elsewhere. However, a small peak was observed at 5.68 MeV. Repeated measurements of yield in the energy region covering its position showed that the small peak is indeed real. This would imply a weak resonance at this point, although it could have been produced by interference effects caused by superposition of the two large resonances below and above it. Later we present arguments to show that we are dealing with real resonance and not with interference effects.

For energies above the 5.90-MeV resonance, the excitation curve dips down and then rises. The maximum energy to which this rise was followed was 6.1 MeV. Recently, Adams *et al.*[18] have confirmed a monotonically rising yield.

Twelve angular distributions were measured. Their shapes between 0° and 90° c.m. are given below the excitation curve on Fig. 1. Their energies and the results of least-square fitting to the expression of the form of Eq. (3.2) are given in Table I. Figures 2, 3, and 4 represent angular distributions at 5.36, 5.68, and 5.90 MeV, the three resonance energies.

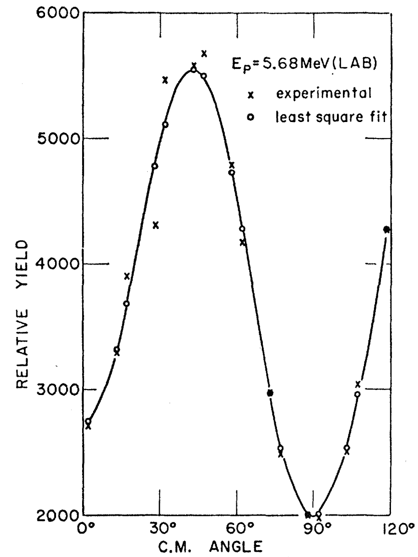

FIG. 3. Center-of-mass angular distribution of 4.43-MeV γ rays from the reaction $C^{12}(p,p'\gamma)C^{12}$, taken at 5.68-MeV (laboratory) bombarding energy.

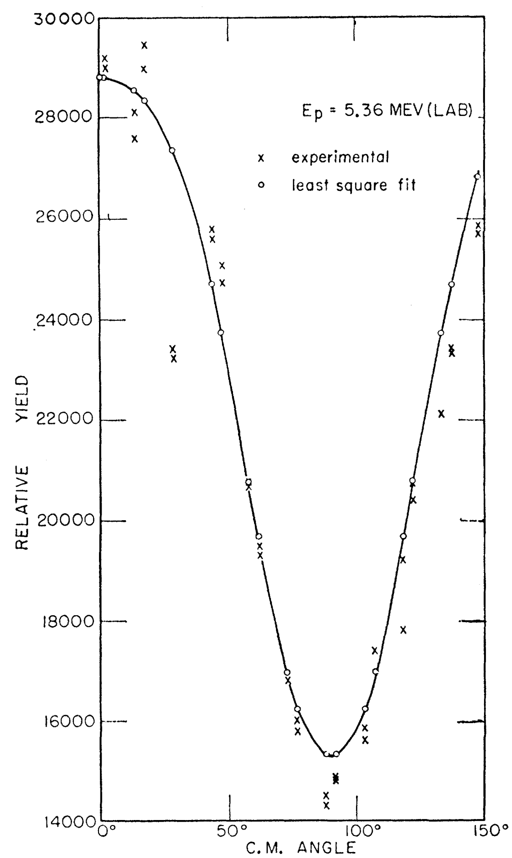

FIG. 2. Center-of-mass angular distribution of 4.43-MeV γ rays from the reaction $C^{12}(p,p'\gamma)C^{12}$, taken at 5.36-MeV (laboratory) bombarding energy.

The most striking feature of all the γ-ray angular distributions is that they are symmetric about 90° in center of mass, which was to be expected since the expression for a pure electric multipole transition will have $A_n \neq 0$ in (3.1) only for even n.[19]

B. Analysis

The expression used is the expression for (p,x,γ) scattering on page 30 of Sharp *et al.*[19] x denotes that the intermediate radiation, in our case the inelastically scattered proton, is not observed.

It is reasonable to expect that for the two resonances of 5.38 and 5.90 MeV the angular distribution will be nearly that of a pure state. Furthermore, since the penetrabilities[20] decrease rapidly with increasing angular momentum of the incoming proton, as can be seen from Table II, and since the energies are very low, it is to be

TABLE II. Penetrability v_l as the function of angular momentum l for reaction $C^{12}+p$.

$E_{lab}=5.6$ MeV	$E_{lab}=0.8$ MeV
$v_0=0.91$	$=0.12$
$v_1=0.77$	$=0.033$
$v_2=0.48$	$=7.1\times10^{-3}$
$v_3=0.18$	$=10^{-4}$
$v_4=0.025$	$=2\times10^{-6}$

[18] H. S. Adams, J. D. Fox, N. P. Heydenburg, and G. M. Temmer, Phys. Rev. 124, 1899 (1961).

[19] W. T. Sharp, J. M. Kennedy, B. J. Sears, and M. G. Hoyle, Atomic Energy of Canada, Ltd., Report CRT-556 (unpublished).

[20] W. T. Sharp, H. E. Gove, and E. B. Paul, Report TPI-70, Chalk River, Ontario (unpublished).

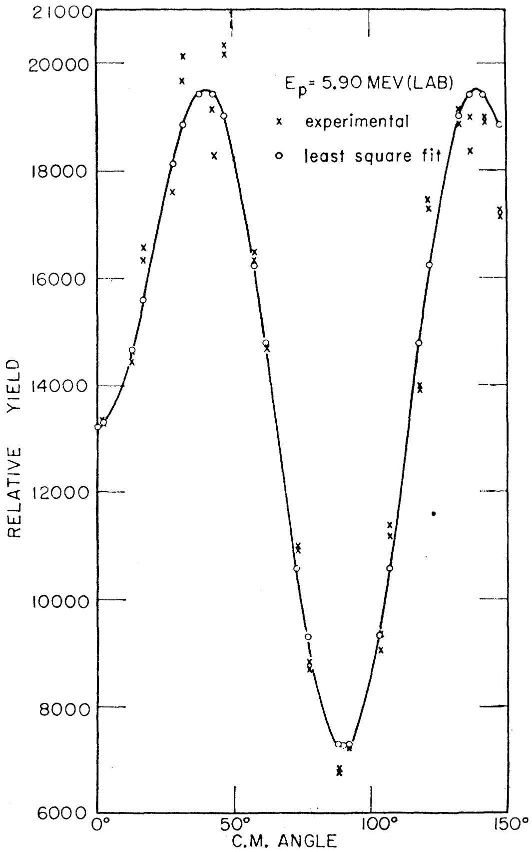

FIG. 4. Center-of-mass angular distribution of 4.43-MeV γ rays from the reaction $C^{12}(p,p'\gamma)C^{12}$, taken at 5.90-MeV (laboratory) bombarding energy.

expected that only the lowest angular momenta of the incoming protons that will give an agreement with experimental facts should be considered.

There are two possible assignments for the angular momentum of the incoming proton which will fit the angular distribution at 5.38 MeV to within 10%. They are $p_{3/2}$ $(3/2^-)$ and $d_{3/2}(3/2^+)$.[21] Both of them require a pure state angular distribution of the form

$$W=P_0+1/2P_2. \tag{3.5}$$

This fixes the spin of the 5.38-MeV (bombarding energy) state in N^{13} as 3/2, but leaves the parity undetermined. Similarly, for the resonance at 5.90 MeV, there are again two possible choices of angular momentum of the incoming proton that will fit the experimental data equally well, namely $d_{5/2}(5/2^+)$ and $f_{5/2}(5/2^-)$, both of which require the angular distribution to be

$$W=P_0+0.571P_2-0.571P_4. \tag{3.6}$$

This fixes the spin of the 5.90-MeV state as 5/2, but again leaves the parity undetermined.

[21] The letter denotes the angular momentum in the usual sense (e.g., $p\rightarrow l=1$). The notation in the bracket denotes the spin and parity of the resonance (J^π).

One might be tempted to resolve the question of parity by considering the shape of the angular distributions in the off-resonance regions where interference between the states is exhibited. Since all angular distributions are symmetric around 90° c.m., one might conclude that all states must have the same parity. The angular distribution below the 5.38-MeV resonance includes the effects of interference between that resonance and the one at 4.81 MeV which is known to have positive parity.[12] Thus all states would be assigned even parity.

Such a conclusion, however, is erroneous. The expression for angular distribution for mixed states[19] still requires that the coefficients of the odd index Legendre polynomials be zero. Hence, no matter what the parity of interfering states, the angular distribution will be symmetric around 90° c.m. Independent means are required to determine the parities.

All the considerations until now were done under the assumption that we deal with only two resonances. But, as we have seen, there is an indication for existence of a small resonance at 5.68 MeV (bombarding energy). Even if this is a real state and is not just produced by interference of the two strong states, it is rather weak and there is no hope to have a pure, or predominantly pure, state angular distribution at this point of the excitation curve. However, if the weak resonance at 5.68 MeV (bombarding energy) is real, then the angular

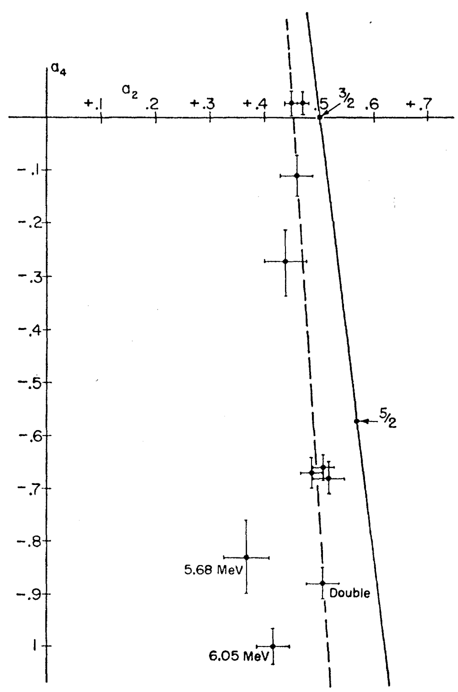

FIG. 5. Dependence of fourth on second-order Legendre polynomial intensity coefficients (Table I) in the angular distributions of 4.43-MeV γ rays from the reaction $C^{12}(p,p'\gamma)C^{12}$, over the range of laboratory bombarding energies from 5.25 to 6.05 MeV. The most general result of the superposition of $3/2\pm$ and $5/2\pm$ resonances involves a straight line dependence of definite slope.

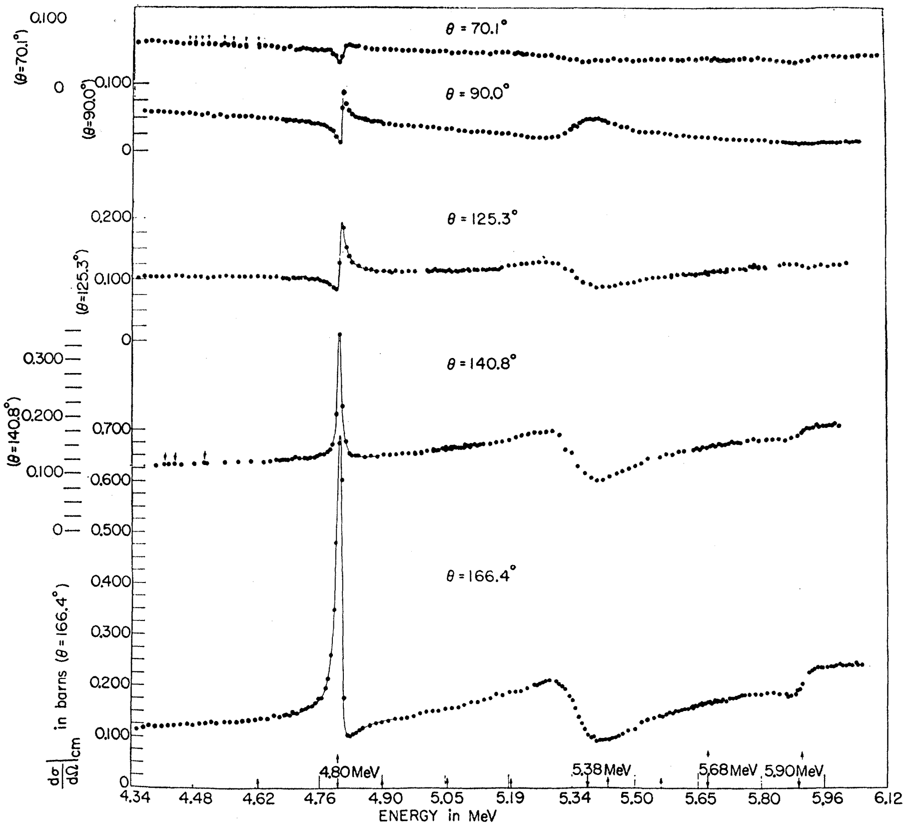

FIG. 6. Dependence on laboratory bombarding energy of the center-of-mass differential cross section at various center-of-mass angles for the elastic scattering of protons from C^{12}. The positions of the resonances observed in the 4.43-MeV γ ray and in the inelastic proton scattering excitation functions are indicated by arrows above the abscissa.

distribution at that point could not be explained by superposition of the two large resonances.

If one assumes the two resonances at 5.38 and 5.90 MeV to be $3/2\pm$ and $5/2\pm$, respectively, then the most general mixed angular distribution can be written in the form of Eq. (3.2) where all the possible values of coefficient a_2 and a_4 have to satisfy the condition[22]

$$a_4 = -8a_2 + 4, \tag{3.7}$$

a straight line in an a_4 versus a_2 diagram.

If we now plot the experimental values of a_4 versus a_2 from Table I, a straight line indeed results, as can be seen from Fig. 5 (broken line), but it does not coincide with the line predicted by Eq. (3.7) (full line through the two points marked 3/2 and 5/2, which are positions of the pure states). The line is slightly shifted. This means that the angular distribution cannot be explained by assuming the superposition of the two states only (3/2, 5/2), although the influence of the other state or states that have to be taken into account to be able to explain the shift is, in general, small.

However, for two points, the 5.68 and 6.05 MeV, the shift is much larger than for the rest. An admixture that would bring the line through all the other points would, admittedly, decrease the discrepancy for these two points; such a transformation would shift the full line in Fig. 5 to coincide with the broken line, but 5.68- and 6.05-MeV points would still not be on the line.

One can assume that there is a state somewhere above 6 MeV which interferes with 5.90-MeV point, but whose influence does not reach low enough to influence the points immediately below it. This explanation is supported by the fact that above 6 MeV there is a rising yield on which the rest of the resonant structure is superimposed.[18]

But this explanation cannot be applied to the 5.68-MeV point, since it is flanked on both sides in energy by points that are much closer to the theoretical straight line. In other words, apart from the states that have to be taken into account to explain the other angular distributions, an additional very narrow state has to be added to fit the angular distribution at 5.68 MeV.

The questions of spin and parity assignment for the 5.68-MeV resonance cannot be answered using γ-ray angular distributions only. However, some general arguments can be made.

The 5.68-MeV resonance cannot be 3/2 or 5/2 because such assignments would not remove the corresponding value of (a_2,a_4) from the general trend. It can be a 1/2 state. Besides that, states produced with higher values of angular momentum have to be considered.

[22] Complete derivation of this equation and examination of possible influence of mixed parities can be found in N. M. Nikolic, Ph.D. dissertation, Columbia University, 1962 (unpublished).

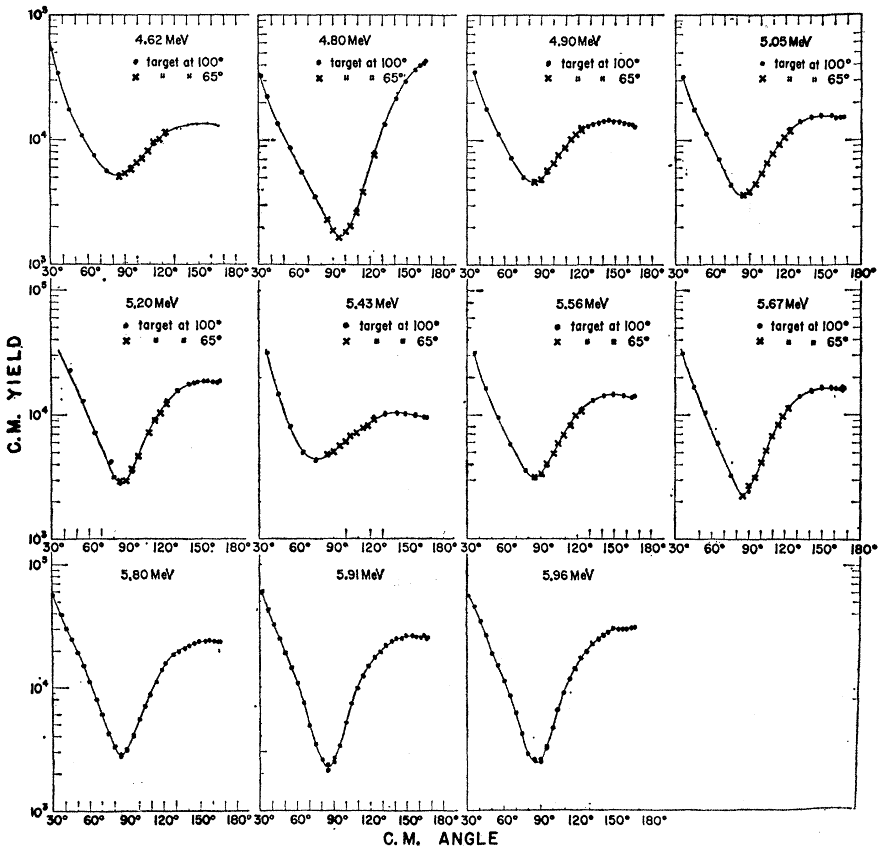

FIG. 7. Center-of-mass angular distributions of protons elastically scattered from C^{12} at various laboratory bombarding energies.

To put an upper limit on the possible values of angular momentum of incoming proton, the Wigner limit rule[23] was used. If the reduced width for channel s and angular momentum l is denoted by γ_{ls}^2, then for protons on C^{12} (reduced mass $\mu=1.54\times10^{-24}$ g, and interaction radius $a=4.95\times10^{-13}$ cm)

$$\sum\gamma_{ls}^2\leq 3\hbar/2\,\mu a=1.36\times10^{-12}\text{ MeV cm.} \quad (3.8)$$

On the other hand,

$$2k\gamma_{ls}^2/A_l^2=\Gamma_{ls}, \quad (3.9)$$

where k is the wave number of the incoming particle, A_l^2 is the reciprocal value of the penetrability, and Γ_{ls} is the center-of-mass width of the resonance for channel s. Then, for an angular momentum to be possible,

$$A_l^2\Gamma_l/2k\leq 1.36\times10^{-12}\text{ MeV cm} \quad (3.10)$$

has to be satisfied.

Taking the value for $\Gamma_{5.68}=0.009$ MeV from the inelastic scattering data (Sec. 5), and the values for A_l^2 from Ref. 20, the highest l turns out to be $l=4$ for which at 5.68 MeV

$$A_4^2\Gamma_4/2k=1.55\times10^{-12}\text{ MeV cm.}$$

Hence, the possible spin and parity assignments for the resonance at 5.68 MeV (bombarding energy) are $1/2\pm$, $7/2\pm$, $9/2^+$.

[23] T. Teichmann and E. Wigner, Phys. Rev. **87**, 123 (1952).

4. ELASTICALLY SCATTERED PROTONS

A. Results

The differential cross section for elastically scattered protons as a function of bombarding energy was measured. The angles in the center of mass coordinate system were chosen in such a way as to make some of the first few Legendre polynomials zero. One exception is $\theta=166.4°$, which was the largest angle we could obtain. At $\theta=70.1°$, $P_4=0$; at $\theta=90.0°$, all odd P's are zero; at $\theta=125.3°$, $P_2=0$; and at $\theta=140.8°$, $P_3=0$. The results are represented in Fig. 6. Statistical errors were 1% or less, and re-running gave reproduction to better than 0.5%. In the region of overlap our results agree completely with those obtained earlier at Rice University.[12]

Angular distributions were also taken at various energies. These results are represented in Fig. 7. The data were transformed to the center-of-mass system, but the curves are not theoretical fits.

B. Analysis

To analyze the data we use the expression for elastic cross section for spin 1/2 projectiles on spin 0 targets when there are other modes of reaction besides elastic scattering; in our case inelastic scattering. Because of the presence of other modes of reaction, this expression contains ratios of partial widths. It is[12,24–27]

$$\frac{d\sigma}{d\Omega}=\frac{1}{k^2}\Bigg|-\frac{\eta}{2}\csc^2\frac{\theta}{2}\exp\left[i\eta\ln\left(\csc^2\frac{\theta}{2}\right)\right]$$
$$+\sum_{l=0}^{\infty}(l+1)P_l(\cos\theta)\exp(2i\omega_l)\left\{\sin\phi_l e^{i\phi_l}+\frac{\Gamma_{l\epsilon}}{\Gamma_l}\sin\delta_l^+\right.$$
$$\left.\times\exp[i(\delta_l^++2\phi_l)]\right\}+\sum_{l=0}^{\infty}lP_l(\cos\theta)$$
$$\times\exp(2i\omega_l)\left\{\sin\phi_l\exp(i\phi_l)+\frac{\Gamma_{l\epsilon}}{\Gamma_l}\sin\delta_l^-\right.$$
$$\left.\times\exp[i(\delta_l^-+2\phi_l)]\right\}\Bigg|^2+\frac{1}{k^2}\Bigg|\sum_{l=0}^{\infty}\frac{dP_l(\cos\theta)}{d\theta}$$
$$\times\exp(2i\omega_l)\left\{\frac{\Gamma_{l\epsilon}}{\Gamma_l}\sin\delta_l^-\exp[i(\delta_l^-+2\phi l)]\right.$$
$$\left.-\frac{\Gamma_{l\epsilon}}{\Gamma_l}\sin\delta_l^+\exp[i(\delta_l^++2\phi_l)]\right\}\Bigg|^2, \quad (4.1)$$

where $\eta=Z_1Z_2l^2/v$, v being the relative velocity and Z_1 and Z_2 atomic numbers of the projectile and target; $k=\mu v/\hbar$, μ being the reduced mass; $\omega_0=0$,

$$\omega_l=\sum_{n=1}^{l}\arctan\frac{\eta}{n};$$

ϕ_l is the potential phase shift; a the interaction radius $=1.5[12^{1/3}+1]\times10^{-13}=4.95\times10^{-13}$ cm; Γ_l= total width of the resonance labeled by l whose partial elastic width is $\Gamma_{l\epsilon}$; and $\delta_l^\pm$ are parallel and antiparallel phase shifts.

The first part of Eq. (4.1) is the coherent scattering. It contains the Coulomb scattering as well, and describes the scattering without a change in relative directions of incoming proton spin and angular momentum. The second part is the spin-flip part, describing the scattering during which the relative direction of spin and angular momentum reverses.

The graphical method of application of Eq. (4.1) is described in Laubenstein and Laubenstein.[28] Such a method is very well suited to analysis of isolated resonances. In the case of interfering resonances, the graphical method becomes too complicated; however, sufficient information for our purposes can be extracted by this method.

Let us begin by considering all resonances as isolated. For a particular resonance corresponding to a definite l, either (but not both) δ_l^+ or δ_l^- will be appreciably different from zero. Thus, only $\sin\delta_l^+$ (or $\sin\delta_l^-$) will be effective in determining the spin-flip term in Eq. (4.1). Under this condition, there could be no interference in the spin-flip term. (It should be noted that this restriction does not necessarily apply to the overlapping resonances.) Therefore, for an isolated resonance, destructive interference can be produced only by interference between Coulomb (described by η), hard sphere (described by ϕ_l's), and resonant (described by δ_l's) scattering.

Furthermore, at scattering angles close to 0° or 180°, the spin-flip term can be neglected because:

$$dP_l(\cos\theta)/d\theta=\sin\theta[dP_l(\cos\theta)/d(\cos\theta)] \quad (4.2)$$

and $\sin\theta$ is very small there. Then, from Eq. (4.1),

$$k\left\{\left[\frac{d\sigma(E)}{d\Omega}\right]_{\max}^{1/2}-\left[\frac{d\sigma(E)}{d\Omega}\right]_{\min}^{1/2}\right\}=\left.\begin{matrix}l+1\\ l\end{matrix}\right\}|P_l|\frac{\Gamma_{l\epsilon}^\pm}{\Gamma_l}, \quad (4.3)$$

where the factor $l+1$ or l corresponds to Γ^+ or Γ^-, respectively.

On the other hand, for angles for which the resonant term is zero (due to corresponding P_l being zero), the resonance is produced only by the spin-flip term, and, if we denote the hard sphere and Coulomb part of resonant term by $|R|^2$, we can write:

$$\left\{k^2\left[\frac{d\sigma(E)}{d\Omega}\right]_{\max}-|R|^2\right\}^{1/2}-\left\{k^2\left[\frac{d\sigma(E)}{d\Omega}\right]_{\min}-|R|^2\right\}^{1/2}=\left|\frac{dP_l}{d\theta}\right|\frac{\Gamma_{l\epsilon}^\pm}{\Gamma_l}, \quad (4.4)$$

where again + or − sign is used to denote the relative direction of spin and orbital angular momentum.

Both Eqs. (4.3) and (4.4) can be used to evaluate $\Gamma_{l\epsilon}/\Gamma_l$. However, $\Gamma_{l\epsilon}/\Gamma_l$ is a function of energy, and so $\Gamma_{l\epsilon}/\Gamma_l$ calculated from Eqs. (4.3) and (4.4) will be only approximately (within 10–20%) the true value at the resonant energy.

5.38 MeV. As has been shown, this should be either a $3/2^-$ or a $3/2^+$ resonance. The differential cross section for a $3/2^-$ resonance observed at 90° c.m. angle should exhibit first constructive and then destructive interference (bottom of Fig. 8). On the other hand, for a $3/2^+$ resonance the differential cross section should exhibit first destructive and then constructive interference. The experimental points shown on top of Fig. 8 have first a dip and then a rise. Therefore, at 5.38 MeV (bombarding energy) we have a $3/2^+$ resonance.

At 90° c.m. the spin-flip term is then zero, so that we

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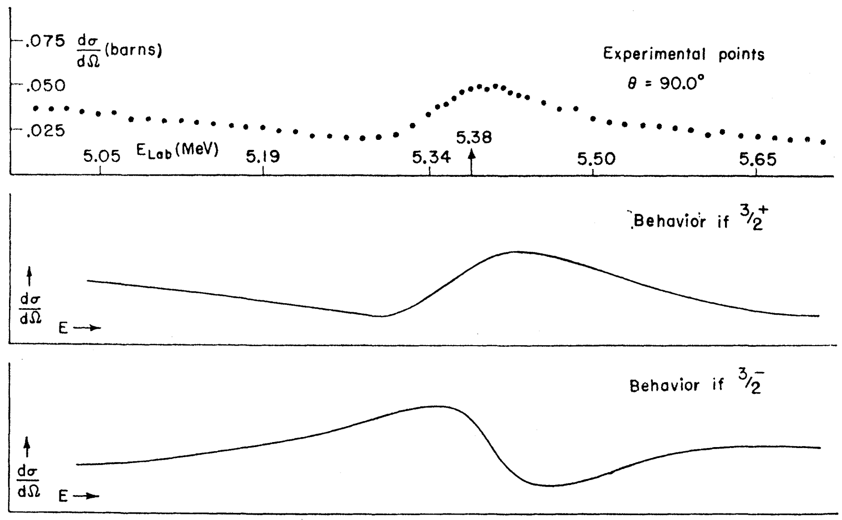

FIG. 8. Experimental and theoretical slopes of the 90° c.m. differential cross section for the elastic scattering of protons from C^{12}, showing the interference effects predicted on different assumptions as to the 5.38-MeV resonance spin and parity.

can use Eq. (4.3) to calculate the ratio of widths. The result is $\Gamma_{2\epsilon}/\Gamma_2=0.393$. At 166.4° c.m. the spin-flip term is very small [Eq. (4.2)], and we can again use the Eq. (4.3) with the result of $\Gamma_{2\epsilon}/\Gamma_2=0.410$ in agreement with the value obtained at 90° c.m.

5.68 MeV. There is only slight evidence for resonant structure at $\theta=70.1°$. As mentioned above, there is a theoretical possibility of this being a $7/2^+$ resonance. If we assume that this is so, then we can use Eq. (4.4) to estimate the ratio of widths. The result is $\Gamma_{4\epsilon}/\Gamma_4=0.036$ which, since Γ_4 is small, as will be seen in the next section, makes $\Gamma_{4\epsilon}$ very small indeed. This explains the fact that we do not see the resonance at different angles. That we see it at all at 70.1° is due to the fact that $dP_4/d\theta$ is fairly large there.

However, all this does not resolve the difficulty of spin and parity assignment for 5.68-MeV resonance. For example, everything that was said above could equally well be applied to a $9/2^+$ assignment. On the other hand, if we assume $1/2^+$ resonance we would get $\Gamma_{0\epsilon}/\Gamma_0=0.043$ using Eq. (4.3). This is of the same order of magnitude as that for $7/2^+$ assignment. The only relevant conclusion that can be drawn from elastic scattering data is that the partial elastic width at 5.68 MeV is very small.

5.90 MeV. The analysis of this resonance is difficult, because it is superimposed on a broad resonance above 6 MeV. Still, the shape of the resonance at different angles can be discerned by extrapolating the general trend of the curve over the resonance (see top of Fig. 9). In this way, it can be seen that at 166.4° the resonance has first a dip and then a rise. This shape agrees with the assignment $5/2^-$, which exhibits first destructive and then constructive interference at this angle; whereas a $5/2^+$ assignment would produce first a rise and then a dip (Fig. 9). Similarly, at $\theta=125.3°$ the shape consists of a rise followed by a dip (Fig. 6), and a $5/2^-$ resonance at this angle has just such a shape.

Furthermore, using Eq. (4.3) at 166.4° we get for a $5/2^-$ assignment $\Gamma_{3\epsilon}/\Gamma_3=0.125$, and for $5/2^+$ assignment $\Gamma_{2\epsilon}/\Gamma_2=0.102$. Now for $5/2^-$ assignment we can use Eq. (4.4) at $\theta=140.8°$ where $P_3=0$. It gives us $\Gamma_{3\epsilon}/\Gamma_3=0.093$, in good agreement with the same ratio calculated at $\theta=166.4°$. On the other hand, for $5/2^+$ assignment, we can use Eq. (4.4) at $\theta=125.3°$ where $P_2=0$. It gives us $\Gamma_{2\epsilon}/\Gamma_2=0.027$ in disagreement with what we obtained at $\theta=166.4°$.

5. INELASTICALLY SCATTERED PROTONS

A. Results

The excitation curves for inelastically scattered protons were measured at two angles, corresponding to center of mass angles of $\theta=54.8°$ (49.1° lid angle) and 140.8° (146.1° lid angle) at the resonant bombarding energy of 5.68 MeV. The first angle is a zero of P_2 and the second of P_3.

These excitation curves are shown in Fig. 10. The errors vary as a function of angle. First, the yield varies, changing the statistical error. In addition, the energy of the scattered protons is smaller for a back angle, other conditions being the same. Thus, it was harder to separate the inelastic peak from the background for back scattering angle, which in turn introduced a larger uncertainty in determining the absolute yield. If 1000 counts are accumulated, the deviation is of the order of 3%. For the back angles, the uncertainty in determining the true number of counts in the peak increased this error to about 5%. However, above 5.40-MeV bombarding energy, the error is mainly produced by statistics. For the forward angle, error is produced only by the statistics, the error of determining the true number of counts in the peak being at most 0.5%.

The weak resonance at 5.68 MeV is worth considering. At $\phi=49.1°(\theta=54.8°)$ the yield is of the order of 1100 ± 33 counts. The statistical error is smaller than

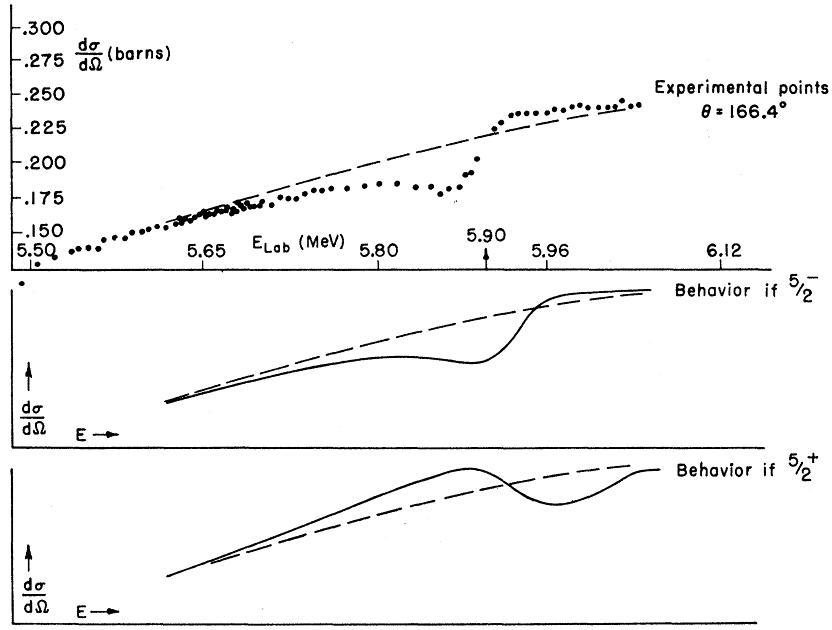

FIG. 9. Experimental and theoretical shapes of the 166.4° c.m. differential cross section for the elastic scattering of protons from C^{12}, showing the interference effects predicted on different assumptions as to the 5.90-MeV resonance spin and parity.

the rise of the excitation function above the general trend at this place. For $\phi=146.1°$ ($\theta=140.8°$) the yield is of the order of 450 ± 21 counts, and the statistical error is exactly the size of the points on the diagram. Thus, in both cases the rise is real and well outside statistical error. This constitutes another and independent confirmation of existence of a small resonance at this energy.

Assuming a one-level Breit–Wigner formula, the estimated full width at half-maximum is $\Gamma_{5.38}=125\pm5$ keV, $\Gamma_{5.68}=10\pm0.5$ keV, and $\Gamma_{5.90}=75\pm5$ keV. Due to the uncertainty of determining background, the estimated error in both $\Gamma_{5.38}$ and $\Gamma_{5.90}$ is of the order of ±5 keV. Due to statistics, the error in $\Gamma_{5.68}$ is 5% or ±0.5 keV.

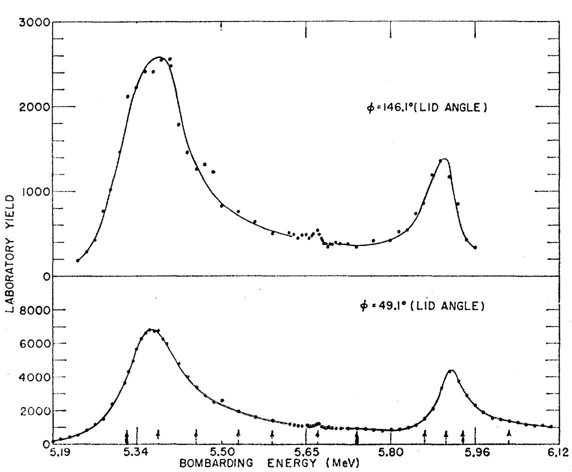

FIG. 10. Excitation functions for the inelastic scattering of protons to the 4.43-MeV state of C^{12}.

The angular distributions of inelastically scattered protons were measured at several bombarding energies. The results are presented in Fig. 11. The least-square fits for all the angular distributions were done on a high speed computer assuming a formula of the type of Eq. (3.1). As has been found by trial, it is enough to take P_n up to and including $n=4$. At the resonances the following angular distribution fits were obtained:

at 5.38 MeV

$$W=(1998\pm24.1)P_0+(193.9\pm36.9)P_1-(112.8\pm54.4)P_2-(271.2\pm69.1)P_3-(30.7\pm77.4)P_4; \quad (5.1)$$

at 5.67 MeV

$$W=(1387\pm17.4)P_0-(218.3\pm25.0)P_1-(14.1\pm37.3)P_2-(565.5\pm47.7)P_3-(306.7\pm53.9)P_4; \quad (5.2)$$

at 5.91 MeV

$$W=(1282\pm11.9)P_0+(337.5\pm20.9)P_1+(360.4\pm28.8)P_2-(429.3\pm36.7)P_3-(87.2\pm38.1)P_4. \quad (5.3)$$

Curves at appropriate energies in Fig. 11 represent these fits.

B. Analysis

Since the two large resonances have been identified, the purpose of the following analysis is to find the parameters for the 5.68-MeV resonance. Using the expression for angular distribution of inelastically scattered protons given in Sharpe *et al.*,[19] p. 16, it is possible to calculate the angular distribution assuming different assignments for the 5.68-MeV resonance.

Thus, if we assume that the 5.68-MeV resonance is

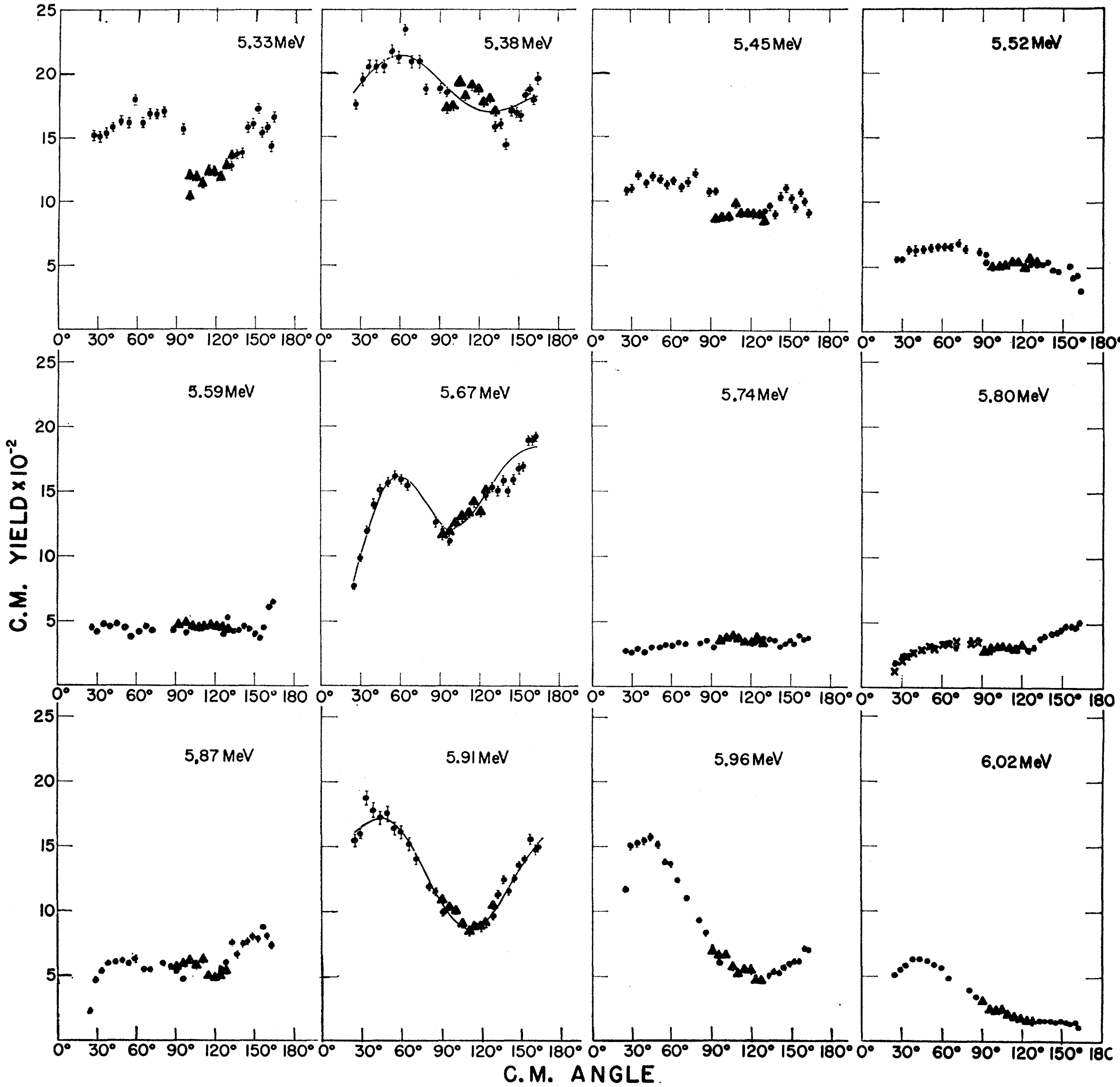

FIG. 11. Center-of-mass angular distributions at various bombarding energies of protons inelastically scattered to the 2^+ first excited state of C^{12} at 4.43 MeV.

$1/2^+$ and calculate the three pure ($3/2^+$, $1/2^+$, $5/2^-$) and the three mixed ($3/2^+$ $1/2^+$, $3/2^+$ $5/2^-$, $1/2^+$ $5/2^-$) terms, the resultant angular distribution should be of the form

$$W = AP_0 + BP_1 + CP_2. \tag{5.4}$$

The same form for angular distribution follows if we assume that 5.68-MeV resonance is a $1/2^-$ or $7/2^-$. Since it is necessary to use all P's up to and including P_4 [see Eqs. (5.1) to (5.3)] to fit experimental results, the resonance at 5.68 MeV cannot be $1/2^\pm$ or $7/2^-$. This leaves only two possibilities, $7/2^+$ and $9/2^+$, both of them formed by an incoming proton of angular momentum $l_1 = 4$.

An assignment of $9/2^+$ would require an angular distribution represented by a sum of Lengendre polynomials up to and including P_8. Since the fits Eqs. (5.1), (5.2), (5.3) are very good indeed (Fig. 11), there is no need to take into account polynomials higher than P_4. Therefore, a $9/2^+$ assignment should be considered as improbable. Hence, we are left with $7/2^+$ assignment as the only possibility.

There are several ways that a $7/2^+$ state in N^{13} can be formed, but they are divided into two groups: one where the orbital angular momentum of the outgoing proton is $l_2 = 2$; the other where $l_2 = 4$. In Sec. 3 the Wigner limit Eq. (3.10) has been used to eliminate some angular momenta of incoming protons. Since the same argument applies here, and the outgoing center of mass energy is ~1 MeV, we have from Eq. (3.9):

for $l_2=2$

$$A_2^2\Gamma_2/2k=3.2\times10^{-13}\text{ MeV cm};$$

for $l_2=4$

$$A_4^2\Gamma_4/2k=1.1\times10^{-9}\text{ MeV cm}.$$

Therefore, $l_2=4$ can be disregarded since it yields a result outside the Wigner limit Eq. (3.10).

Retaining $l_2=2$, the angular distribution resulting from a superposition of the three states $3/2^+$, $7/2^+$, and $5/2^-$ can be expressed in general as

$$W=\sum_{n=0}^{4} a_nP_n \tag{5.5}$$

in agreement with Eqs. (5.1), (5.2), and (5.3).

Not only does the general shape of $7/2^+$ angular distribution agree with the experimental results, but the detailed behavior of the coefficients a_n is in agreement with the experiment as we now show.

Each pure and interference angular distribution can be expressed as a sum of Lengendre polynomials. Let us denote the intensity coefficients in pure $3/2^+$ distribution as B_i, in $5/2^-$ as C_i, and in $7/2^+$ as D_i, and let the corresponding amplitudes be s, p, and q, respectively; s, p, q are chosen in such a way that s is real, and the phases of p and q are measured with respect to s. Let $3/2^+\,5/2^-$ interference term have coefficients A_{bci}, $3/2^+\,7/2^+$ A_{bdi}, and $5/2^-\,7/2^+$ A_{cdi}. Then, in general, the coefficients in Eq. (5.5) have the following form:

$$\begin{aligned}
a_0&=s^2B_0+|p|^2C_0+|q|^2D_0,\\
a_1&=s(p+p^*)A_{bc1}+(pq^*+p^*q)A_{cd1},\\
a_2&=|p|^2C_2+|q|^2D_2+s(q+q^*)A_{bd2},\\
a_3&=(pq^*+p^*q)A_{cd3},\\
a_4&=|q|^2D_2.
\end{aligned}\tag{5.6}$$

Equations (5.6) were obtained by calculating the actual angular distributions and then extracting the general form containing all possible combinations. A's, B's, C's, and D's can be either positive or negative (B_0, C_0, and D_0 are always positive), and they range in value between 1 and 10.

To compare Eq. (5.6) with experimental results (5.1), (5.2), and (5.3), it is necessary to have a criteria for when one of a_i can be considered negligible. It is reasonable to assume that whenever one of Δa_i is almost as large, if not larger, as the corresponding a_i, then this a_i can be considered negligible. This is because the errors Δa_i in (5.1), (5.2), and (5.3) are all of the same order of magnitude. Hence, whenever $a_i\approx\Delta a_i$, this happens because a_i becomes unusually small, and not because Δa_i becomes unusually large.

At 5.38 MeV, s is large, being the amplitude of $3/2^+$ resonance. Therefore a_0, a_1, and a_2 should be large [a_2 does not necessarily have to be large because there might occur cancellations in third Eq. (5.6)]. As can be seen from Eq. (5.1), this condition is satisfied.

At 5.67 MeV, $|q|$ is large, so all the terms in Eq. (5.2) should be large (a_1 and a_2 need not be because of cancellations). This condition is satisfied. More important, at this energy a_4 has to be substantial because of the presence of $|q|^2$ in last Eq. (5.6), which it is. In addition, the a_4's at 5.38 and 5.91 MeV are negligible, which they should be since the 5.68-MeV resonance is very narrow.

Finally, at 5.90 MeV from Eq. (5.6) all a's except a_4 should be large and they are. Thus, the $7/2^+$ assignment for the 5.68-MeV resonance is in very good agreement with experimental results and should be considered as almost certain.

Furthermore, the foregoing analysis strengthens the $5/2^-$ assignment of the 5.90-MeV resonance. However, let us examine the possible assignment of $5/2^+$. Then the pure state angular distribution would be $W\sim P_0$. The interference terms would in that case contain a Clebsch–Gordan coefficient of the form $(0000|n0)$ which is different from zero only for $n=0$. So the interference term is also isotropic. This means that a_2 in Eq. (5.1) should be very small. But this is not so. Hence, the experimental evidence from inelastic scattering contradicts the $5/2^+$ assignment for the 5.90-MeV resonance.

6. CONCLUSION

Three resonances were observed in the bombarding energy range 5–6 MeV.

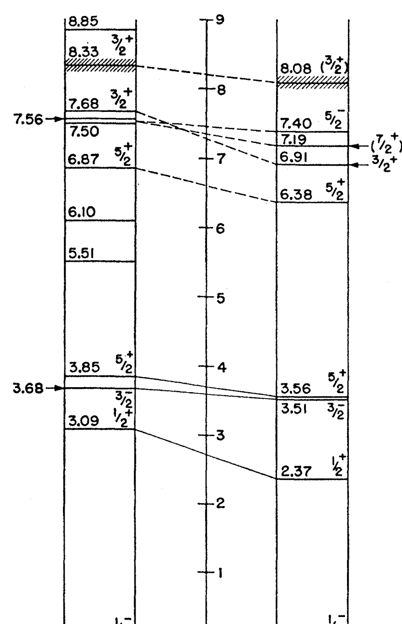

FIG. 12. Comparative scheme of known energy levels of the two members of the $A=13$ isotopic spin doublet.

At 5.38-MeV bombarding energy, which corresponds to 6.91-MeV excitation energy in N^{13}, the resonance has $j^\pi = 3/2^+$ ($d_{3/2}$). Its total width in the laboratory is 125 ± 5 keV, or 115 ± 5 keV in the center-of-mass system. This implies an elastic width of 50 keV in the laboratory system, or 46 keV in the center-of-mass system.

At 5.68-MeV bombarding energy, which corresponds to 7.19-MeV excitation energy in N^{13}, the resonance has very probably, but not certainly, $j^\pi = 7/2^+$ ($g_{7/2}$). Its total width in the laboratory system is 10 ± 0.5 keV, or 9 keV in the center-of-mass system. The elastic width is 0.4 keV, or 0.36 keV in the center-of-mass system.

At 5.90-MeV bombarding energy, which corresponds to 7.40-MeV excitation energy in N^{13}, the resonance has $j^\pi = 5/2^-$ ($f_{5/2}$). Its total width in the laboratory system is 75 ± 5 keV, or in the center-of-mass system 69 ± 5 keV. This corresponds to an elastic width of 7.5 keV in the laboratory system, or 6.9 keV in the center-of-mass system.

The intermediate coupling shell-model calculations for an $A = 13$ nucleus require a $7/2^+$ state at an excitation energy of ~ 7.5 MeV and a $5/2^-$ state at an excitation energy of 4 to 5 MeV. We have identified a state at 7.19 MeV that has all the properties consistent with a spin parity assignment of $7/2^+$. Therefore the existence of the $7/2^+$ state as calculated theoretically should be considered verified experimentally with a high degree of probability.

In contrast, the $5/2^-$ assignment for the 7.40-MeV state is certain; however, its energy is quite high when compared with the energy that is predicted for it.This might indicate that the choice of $L/K = 6.8$ that was made has to be changed so as to get an agreement between experiment and calculation.

It is interesting to compare the level scheme of N^{13} with its isotopic spin doublet (mirror nucleus) C^{13} for excitation energies below 8–9 MeV. In Fig. 12[29] the ground states are at zero energy.

The correspondence between the first three excited levels in both nuclei is indicated on the diagram by connecting lines. The Coulomb correction to the energy of N^{13} levels relative to C^{13} levels is[30]

$$\Delta = 6Ze^2/5R,$$

where Z and $Z+1$ are the numbers of protons in mirror nuclei. In our case this is expected to be of the order of 20–30 keV. All three of these levels are single-particle levels.[29]

Above this, we have the corresponding levels connected by broken lines. The exact correspondence of 7.19- and 7.40-MeV levels in N^{13} with the 7.50- and 7.56-MeV levels in C^{13} remains to be determined. What seems to be fairly certain is that these two pairs correspond to each other.[6] Since at these energies we are no longer dealing with single-particle excitation states, one need not expect that the order of levels remains the same in both nuclei. Indeed there is almost certainly a crossover by $3/2^+$ state at 6.91 (N^{13}) and 7.68 (C^{13}) MeV.

There are two levels in C^{13}, at 5.51- and 6.10-MeV excitation, that have no known corresponding levels in N^{13}. They have been observed in $B^{11}(He^3,p)C^{13}$ reaction.[16] It has been estimated that the upper limit for their laboratory total width should be 10 keV. If they are indeed real, this fact explains why they have not been observed in N^{13} in $C^{12}+p$ reactions.

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